

## THE STEADY STATE ICE LAYER PROFILE ON A CONSTANT TEMPERATURE PLATE IN A FORCED CONVECTION FLOW—II. THE TRANSITION AND TURBULENT REGIMES

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**Abstract**—The process of transition from laminar to turbulent flow on an ice surface was found to be very significantly different from that on a flat plate. The influence of the ice surface occurs because the shape of ice layer responds to the changes in heat-transfer coefficient that occur in the transition regime. As a result of this interaction two modes of transition were observed. Each mode was associated with a distinctive ice profile shape and Reynolds number for the onset of turbulent heat transfer. For both modes the onset Reynolds number was substantially lower than that for a flat plate. For some experimental parameters an interesting hysteresis phenomenon occurred in which the steady state ice layer could take either one of the two characteristic shapes depending on how equilibrium was approached. The decrease in the ice layer thickness that occurs in the transition region was also observed to have a major effect on the heat-transfer rates in the turbulent regime for some distance downstream of transition.

### NOMENCLATURE

- $H$ , height of ice step in transition region;  
 $L$ , latent heat of fusion of ice;  
 $Nu_x, Nu_H, Nu_{xH}$ , local Nusselt numbers,  
 $hx/\lambda_w, hH/\lambda_w$  and  $hx_H/\lambda_w$ ;  
 $Pr$ , Prandtl number;  
 $Re_x, Re_H, Re_{xH}$ , Reynolds numbers,  
 $U_\infty x/\nu, U_\infty H/\nu$  and  $U_\infty x_H/\nu$ ;  
 $Re_\theta$ , Reynolds number based on momentum  
thickness  $\theta$ ;  
 $T_f, T_w, T_\infty$ , freezing, plate and free stream  
temperatures;  
 $Tu$ , free stream turbulence,  $(\bar{u}^2)^{1/2}/U_\infty$ ;  
 $U_\infty$ , free stream velocity;  
 $h$ , local heat-transfer coefficient;  
 $v$ , migration velocity of "step" transition  
point;  
 $x_H$ , distance from leading edge to position of  
peak heat-transfer coefficient.

### Greek symbols

- $\alpha$ , ice thickness parameter, equation (1);  
 $\delta, \delta_i$ , velocity boundary layer and ice  
thicknesses;  
 $\theta_c$ , cooling temperature ratio  
 $(T_f - T_w)/(T_\infty - T_f)$ ;  
 $\lambda_i, \lambda_w$ , ice and water thermal conductivities;  
 $\nu$ , kinematic viscosity;  
 $\rho$ , density;  
\*, peak value.

### INTRODUCTION

VERY few studies of transition or turbulent flow over a phase change medium have been reported. Most of the studies involving forced convection and phase change, including those referred to in Part I of this

paper have treated the analytically much simpler laminar flow regime. As it was in the laminar regime one would expect that the interaction of the flow and the shape of the phase changes interface will be the interesting aspect of this problem. This interaction might be expected to be even more significant because it is well known that both transition and turbulent flows are strongly affected by the free stream pressure gradient.

Some studies have been reported where this interaction was observed. For example the multi-mode behavior for a water jet penetrating on ice block [1] and the wave-like instabilities observed on the bottom of ice sheets in rivers [2] and in some laboratory ablation simulation tests [3,4] have been attributed to this interaction. In the heat shield ablation studies similar instabilities have been observed which manifest themselves as "cross-hatching" patterns on the ablating surface [5,6]. The ablation problems are; however, somewhat more complex in that they involve three phases—a solid, a liquid film flowing over the solid, and a gas flow over the liquid.

To see how this interaction could occur in the present problem of forced convection flow over an ice sheet, the general nature of the flow will first be examined qualitatively. Figure 1 schematically illustrates the flow regimes and the shape that the ice surface has in each regime. The ice shape in the laminar regime was the subject of Part I of this paper. In that regime the ice layer thickness continually increases with distance from the leading edge of the plate due to the decrease in heat-transfer coefficient that occurs as a laminar boundary layer develops. In the transition from laminar to turbulent flow the heat-transfer coefficient undergoes a substantial increase. For a constant temperature plate

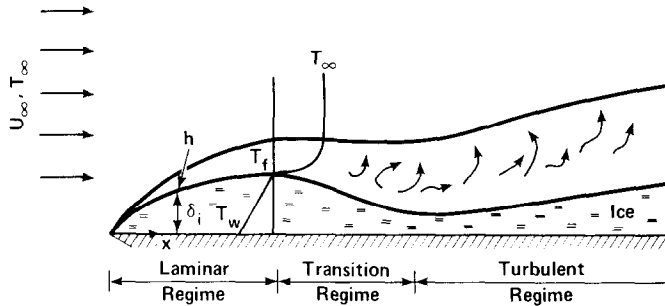


FIG. 1. Schematic representation of the ice profile and the flow regimes.

this must mean a decrease in ice thickness occurs through the transition regime. This decrease in ice thickness and the pressure gradients that result could, potentially, have a major effect on the transition [7].

In the turbulent regime it will be of interest to determine what effect the decrease in ice thickness that occurred in the transition regime will have on the nature of the flow and the magnitude of the heat-transfer coefficients. First; however, the nature of the laminar to turbulent transition on a phase change surface will be examined in detail.

It should be noted that the experimental facility used in these studies was the same as that described in Part I [8] of the paper except that the range of velocities was extended to give Reynolds numbers up to  $7 \times 10^5$  on the 1.52 m. long plate and the turbulence ratios  $\theta_c$  up to 44.

**FLOW TRANSITION ON AN ICE SURFACE**

In studying the transition from laminar to turbulent flow on an ice surface it became apparent that there are actually two different modes by which the

transition can occur. The most obvious difference between the modes was that each was associated with a distinctive shape of the ice surface in the transition regime. The transition regime is defined approximately as the part of the ice profile for which the ice layer thickness decreases with distance along the plate. Figure 2(a) shows the shape of the steady state ice surface for the two modes. These profiles were obtained in a region of velocity and temperature for which either mode could exist. The factors which determine which of the modes actually exists in a given situation will be discussed later.

In Fig. 2(a) it can be seen that the ice thickness for one mode which will be called the "smooth" transition decreases gradually through the transition regime. In the turbulent regime the ice thickness again increases smoothly. In the other mode the

sudden change at transition which is followed by a distinct depression in the ice surface. As might be expected this latter mode is associated with a flow separation at the transition point. A photograph of this transition mode in Fig. 2(b) shows the sharp nature of the break in the ice layer profile at the

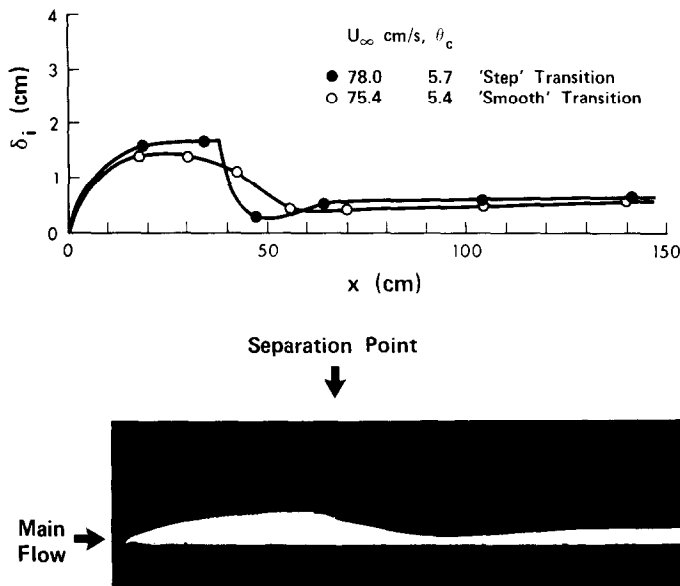


FIG. 2. (a) Characteristic shapes of the ice surface for the "smooth" and "step" transition modes. (b) Photograph of the ice surface for the "step" transition mode,  $U_\infty = 78.0$  cm/s,  $T_i = 1.0^\circ\text{C}$  and  $\theta_c = 16.0$ .

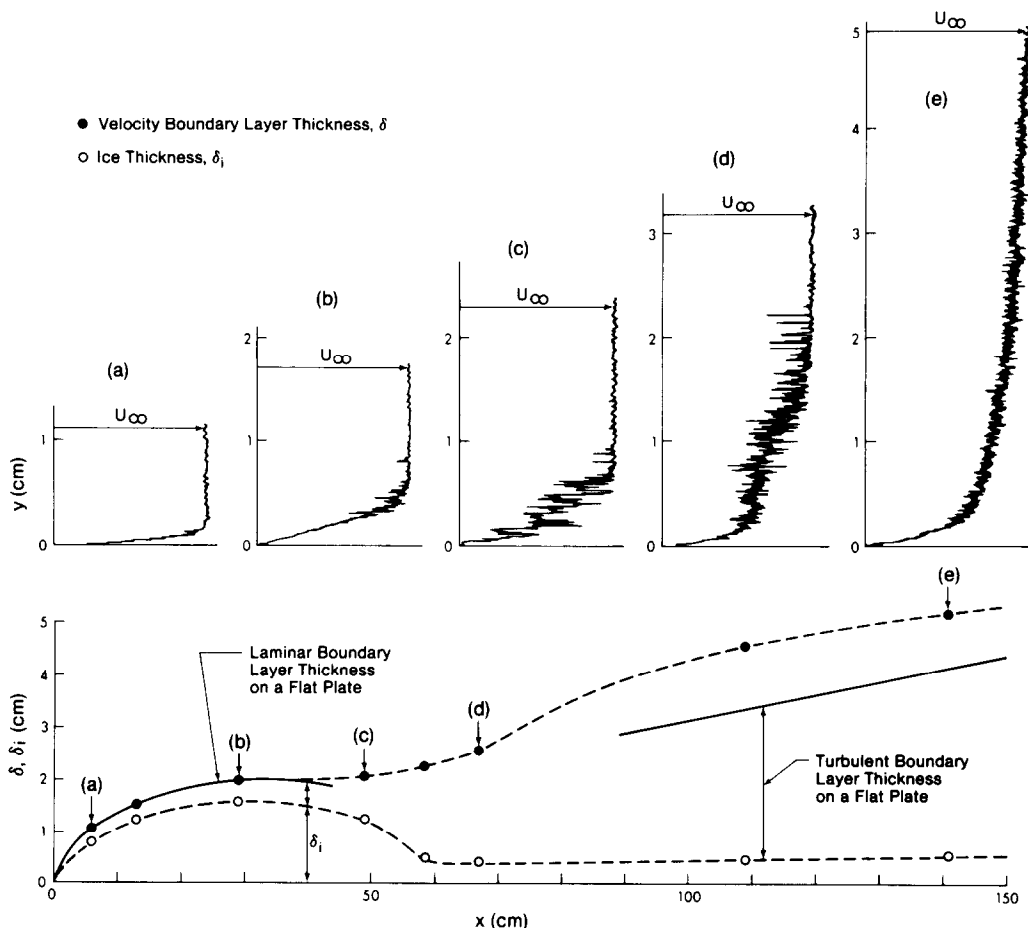


FIG. 3. Velocity boundary layer development and ice layer profile for the "smooth" transition,  $U_\infty = 77$  cm/s,  $T_\infty = 0.7^\circ\text{C}$  and  $\theta_c = 4.9$ .

separation point. The existence of a recirculating separation bubble behind the downstream face of the step was confirmed by the injection of dye into the bubble region. No such separation bubble could be detected when the "smooth" transition occurred. The behavior of each transition mode will now be examined in more detail.

#### THE "SMOOTH" TRANSITION MODE

Figure 3 shows the ice layer profile and the velocity boundary layer development on the ice surface on which a "smooth" transition occurs. For the conditions of this figure, free stream velocity  $U_\infty = 77$  cm/s and cooling temperature ratio  $\theta_c = 4.9$ , the rate of increase of ice layer thickness with  $x$  becomes less than that predicted by laminar theory at about  $x = 25$  cm. At approximately this position an increased level of turbulence was also observed in the boundary layer. This position which corresponds to a Reynolds number of about  $1 \times 10^5$  in the present case will be referred to as the onset point. The ice layer thickness has a maximum value and then decreases with distance from the leading edge. The ice layer thickness decreases from about  $x = 30$  cm to  $x = 60$  cm, that is over the range

$Re_x = 1.3 - 2.7 \times 10^5$ . The velocity profiles measured in this region show an increasing degree of velocity fluctuation in the boundary layer as the transition proceeds. The velocity profiles also show some inflection points. These inflections are shown more clearly in Fig. 4 where mean profiles are plotted for the same conditions as used in Fig. 3. This figure shows that the profile in the laminar regime agrees with the Blasius profile and those far downstream of the transition approach the  $1/7$ th power law for a fully turbulent flow. In the transition regime the profiles are; however, highly distorted. The points of inflection in the profiles in this region are typical of the unstable nature of a flow in an adverse pressure gradient.

The onset Reynolds number that is the Reynolds number at which a measurable effect of turbulence on heat transfer occurred, was about  $3 \times 10^5$  for the cooling plate without an ice cover. The measured free stream turbulence level in the tunnel was rather high at about 0.8–1.4% RMS velocity fluctuation. Considering this turbulence level the onset Reynolds number without ice is consistent with the values suggested by Schlichting [9] of  $3.5 \times 10^5$  to  $1 \times 10^6$  for a turbulence level of 0.5%.

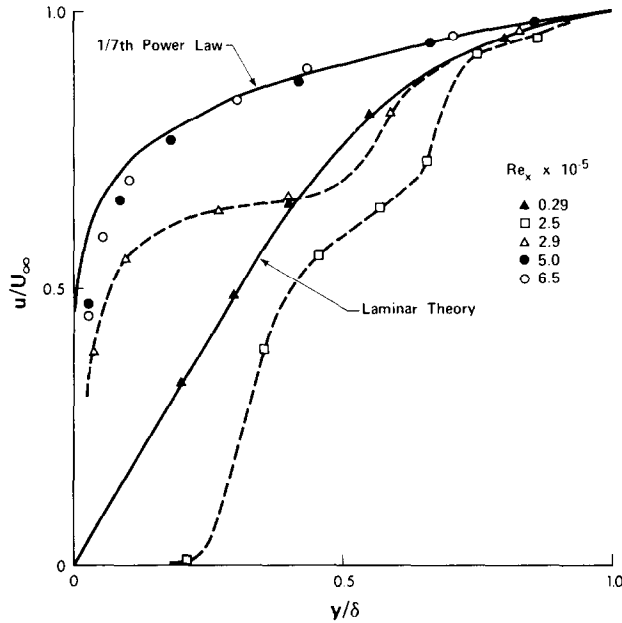


Fig. 4. Normalized mean velocity profiles for the conditions in Fig. 3,  $U_\infty = 77.0$  cm/s,  $T_x = 0.7$  C and  $\theta_c = 4.9$ .

The Reynolds number at which the “smooth” transition begins was measured for several different conditions of velocity and temperature. The onset of the “smooth” transition was found to occur at Reynolds numbers in the range  $7 \times 10^4$  to  $1 \times 10^5$  which is significantly less than the onset Reynolds number for a flat plate with no ice. This reduction of the Reynolds number for flow over an ice surface may result from the interaction of the flow and the ice surface shape that was suggested previously. As turbulence develops in the boundary layer the heat transfer increases and the ice layer thins. The thinning of the ice layer generates an adverse pressure gradient which augments the amplification rate of turbulence in the transition regime [7]. Although direct measurements of ice surface roughness could not be made visual observation of light reflected from the surface would strongly suggest that the surface is indeed very smooth. Surface roughness is not likely, therefore, to be a factor in causing the lower onset Reynolds numbers observed on the ice surface.

The Granville correlation for the transition Reynolds number that is the Reynolds number at which the flow is completely turbulent (see [7]) is

$$Re_\theta(x_{tr}) \approx Re_\theta(x_i) + 450 + 400 e^{6.0 \lambda_m}$$

where  $\lambda_m$  is the mean value of the Pohlhausen integral parameter between the critical Reynolds number,  $Re_\theta(x_i)$ , and the transition Reynolds number,  $Re_\theta(x_{tr})$ . For an adverse pressure gradient  $\lambda_m$  is negative. This means that the transition moves forward toward the initial critical point as the adverse pressure gradient increases. Since the magnitude of the adverse pressure gradient is inversely related to the distance over which the transition

occurs an interaction between the shape of the ice surface and the transition process exists. If the rate of decrease of ice layer thickness in the transition region becomes sufficiently large a flow separation occurs in this region. A “step” transition which will be described in the next section then occurs.

#### THE “STEP” TRANSITION MODE

The “step” transition which proceeds by a flow separation exhibits behavior very different from that for the “smooth” transition. In particular the transition Reynolds number is quite different. A “step” transition normally forms because of a flow separation occurring just past the point of maximum ice thickness on a “smooth” transition ice profile. The initial transition point for the “step” transition is; therefore, close to that for a “smooth” transition. However, if the “step” transition is observed over a period of time it will be seen to migrate very slowly upstream. For example, in one test with  $U_\infty = 87.9$  cm/s and  $\theta_c = 24$  the “step” transition originally formed at a position such that  $Re_x$  was about  $1.5 \times 10^5$ . During a test of three days duration at fixed free stream velocity and temperature conditions the “step” moved toward the leading edge of the plate so that at the end of the test the Reynolds number at the step had decreased to  $5 \times 10^4$ . Because of the flow separation occurring in a “step” transition the flow can change from a laminar flow upstream of the separation to a turbulent flow downstream of the step in a very short distance.

When a flow separation occurred the transition point was then taken to be the same as the point of flow separation. The phenomenon of migration of the “step” transition point toward the leading edge of the plate occurs because turbulence in the separation

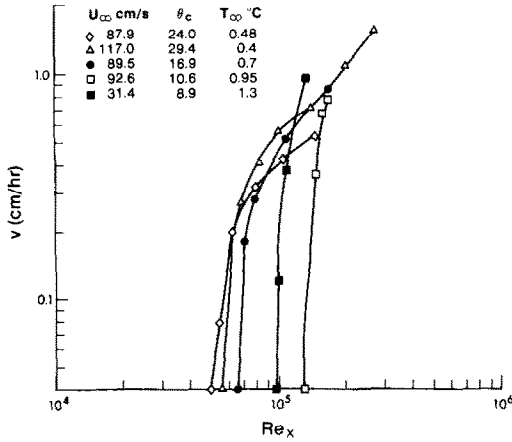


FIG. 5. Velocity of migration of the position of the "step" transition for various conditions.

region produces a high heat transfer coefficient on the rearward face of the "step". This melts away the rearward face of the step, moves the separation point upstream.

Figure 5 shows the migration velocity of the step as a function of the Reynolds number at the separation point for a number of different test conditions. In all cases the transition point moves upstream until it reaches some final Reynolds number position where the migration velocity goes to zero.

It was found that this final Reynolds number for the "step" transition could be related to a parameter,  $\alpha$ , which describes how fast the ice thickness increases with  $x$  in the laminar regime. From Part I of this paper [8] it was shown that the ice layer profile even for large ice layer thicknesses has a very nearly parabolic shape,

$$\delta_i = \alpha \sqrt{x}$$

where

$$\alpha = \frac{\lambda_i}{\lambda_w} \frac{\theta_c}{0.332 Pr^{1/3}} \left( \frac{v}{U_\infty} \right)^{0.5} \quad (1)$$

The Reynolds numbers at the final position of the "step" transition are plotted vs the parameter,  $\alpha$ , in Fig. 6. The range of Reynolds numbers measured at the "step" transition was  $3.9 \times 10^4$  to  $1.9 \times 10^5$  and over this range the results correlated satisfactorily with the empirical expression

$$Re_x = \frac{0.09}{\alpha} 10^5 \quad (2)$$

when  $\alpha$  is expressed in units of  $m^{1/2}$ .

The lower limit of Reynolds number for the "step" appeared to be limited only by the limitations on the thickness of ice that could be grown in the present experiment. As will be seen in the next section the upper limit is determined by the fact that at small values of  $\alpha$  a "smooth" rather than a "step" transition will occur.

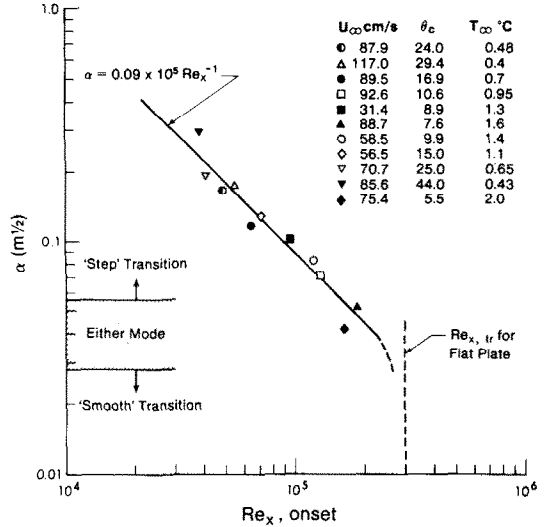


FIG. 6. The correlation of the final steady state position of the "step" transition and demonstration of the approximate ranges of parameter  $\alpha$  for which the "step", "smooth" or either transition mode could exist.

#### THE DETERMINATION OF THE TRANSITION MODE

Generally it was found that the "smooth" transition occurred if the ice layer was thin and the "step" transition occurred if the ice layer was thick. There was, however, a certain range of conditions for which either mode could exist depending on the past history of the ice layer. For example, if the plate temperature is initially only slightly below 0°C the ice layer that forms is thin and a "smooth" transition occurs. At a constant free stream velocity the thickness of the ice layer at the transition point is directly related to the temperature ratio  $\theta_c$ . If the cooling plate temperature is lowered, that is  $\theta_c$  increased the ice layer will become thicker until at some value of  $\theta_c$  a flow separation will occur in the transition regime. For a free stream velocity of 78 cm/s flow separation was observed at  $\theta_c = 7$ . Once flow separation occurs the transition mode spontaneously changes to a "step" transition. Now if the plate temperature is increased back toward 0°C (that is  $\theta_c$  is decreased) the "step" transition continues to exist for values of  $\theta_c$  less than that at which the original separation occurred. For the case of a free stream velocity of 78 cm/s the change back to a "smooth" transition does not occur until  $\theta_c$  is less than about 4. There is therefore some hysteresis involved in determining which of the two types of transition will occur. The bistable region in which either of the two modes can exist is best related to the ice thickness parameter,  $\alpha$ . Figure 6 shows the transition mode which was observed for various ranges of  $\alpha$ . It will be noted that the upper limit of the bistable zone corresponds approximately to the region in which the Reynolds numbers for the "step" and the "smooth" transition are the same.

HEAT-TRANSFER RATES IN THE  
TURBULENT REGIME

The shape of the ice surface in the transition regime has a strong influence on the heat-transfer rates in the turbulent regime. For the case of a "step" transition the problem might be considered to be similar to that for a separated flow associated with a rearward facing step on a plate. For the heat transfer in a separated flow, Seban [10] suggested that the local heat-transfer coefficient downstream of the reattachment might be proportional to the 0.8 power of the free stream velocity. Recently, Kasagi *et al.* [11] reported that in air the peak Nusselt number,  $Nu_H^*$ , is correlated with the 0.77 power of Reynolds number.

That is

$$Nu_H^* = \text{const.} \cdot Re_H^{0.77} \tag{3}$$

where  $Nu_H$  and  $Re_H$  are defined by a step height,  $H$ . Introducing another reference length,  $x_H$  (see Fig. 7), instead of  $H$ , equation (3) can be transformed as follows:

$$Nu_{x_H}^* = \text{const.} \cdot Re_{x_H}^{0.77} \left(\frac{H}{x_H}\right)^{-0.23} \tag{4}$$

Dividing equation (4) by a correlation of local Nusselt number on a flat plate,  $Nu_{x_H} = \text{const.} \cdot Re_{x_H}^{0.8}$ , we obtain

$$\frac{Nu_{x_H}^*}{Nu_{x_H}} = C \cdot \left(Re_{x_H}^{0.13} \frac{H}{x_H}\right)^{-0.23} \tag{5}$$

The LHS term of equation (5) refers to the ratio of the peak Nusselt number for a separated flow to the value for a flat plate.

To produce Fig. 7 the ice thickness just downstream of the transition was measured and used with the equation of one dimensional heat conduction in the ice {equation (2) of Part I [8]}  $\epsilon$

$$Nu_x = \frac{\lambda_i}{\lambda_w} \theta_c \frac{x}{\delta_i} \tag{6}$$

to infer the Nusselt number occurring in this region. The ratio of this measured Nusselt number to the value calculated for a flat plate is plotted against  $Re_{x_H}^{0.13} \cdot H/x_H$ .

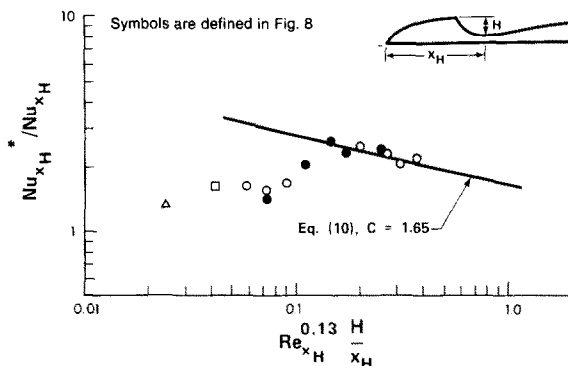


Fig. 7. The peak Nusselt number downstream of transition plotted against a step height parameter  $Re_{x_H}^{0.13} H/x_H$ .

When a "step" transition occurred,  $Re_{x_H}^{0.13} H/x_H > 0.15$ , the data coincide with the values given by equation (5) with constant  $C = 1.65$ . This result supports the argument that the "step" transition acts like a rearward facing step as far as its effect on heat transfer to the ice is concerned. For  $Re_{x_H}^{0.13} H/x_H < 0.15$ , a flow separation was not observed; however, the peak Nusselt number was still larger than that for a flat plate. It is therefore apparent that for either transition mode the heat-transfer coefficient, at least near the transition point on an ice surface, will not be predicted from the results for a flat plate.

In Fig. 8 the values of  $Nu_{x_H}^*/Nu_{x_H}$  are replotted as a function of ice thickness parameter  $\alpha$ . In this figure the effect of the separated flow above  $\alpha = 0.05$  is to increase  $Nu_{x_H}^*$  to a value more than twice the Nusselt number for a flat plate. For  $0.03 < \alpha < 0.05$  the hysteresis behavior mentioned previously occurs. The open circles refer to experiments in which the  $\alpha$  is being increased in steps from zero and the solid points refer to experiments where  $\alpha$  is being decreased in steps from a higher value. The value of  $Nu_{x_H}^*/Nu_{x_H}$  would be expected to approach one as  $\alpha$  approaches zero since in this limit the ice thickness is very small. For the range of velocities and temperature ratios which probably has the most practical interest, the shape of the ice surface in the transition regime does however, have a large effect on the heat transfer downstream of the transition.

Figure 9 summarizes the variation of Nusselt number on an ice surface over a range of Reynolds numbers from the laminar through to the fully developed turbulent regime. From this figure it can be seen that the Nusselt numbers throughout the turbulent regime are substantially larger than those given by the von Kármán correlation for turbulent flow on a flat plate. As was observed previously the "step" transition results in a peak in the Nusselt number occurring just downstream of the transition. However, for both types of transition the effect of the ice shape on the heat-transfer rates persists for some distance downstream of the transition. At the largest Reynolds numbers obtainable in these tests,  $7 \times 10^5$ , the Nusselt number is still about 20% greater than that for a flat plate. This is understandable particularly for the case of a "step" transition where large scale turbulence can be generated in the free shear layer of the separation zone. For a rearward facing step such turbulence has been shown to affect the heat-transfer rates some distance downstream of the reattachment region [12].

CONCLUSIONS

The profile of the steady state ice layer on a cold plate in a forced convection flow is effected by a number of factors. This study was primarily a phenomenological investigation aimed at identifying the important factors in the transition and turbulent regimes.

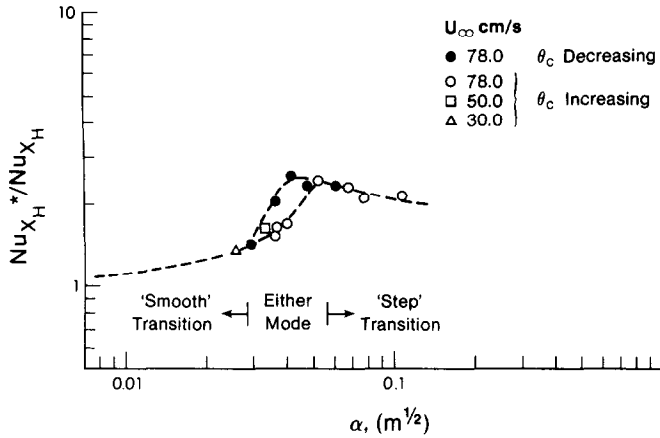


FIG. 8. The peak Nusselt number downstream of transition plotted against the ice thickness parameter  $\alpha$ .

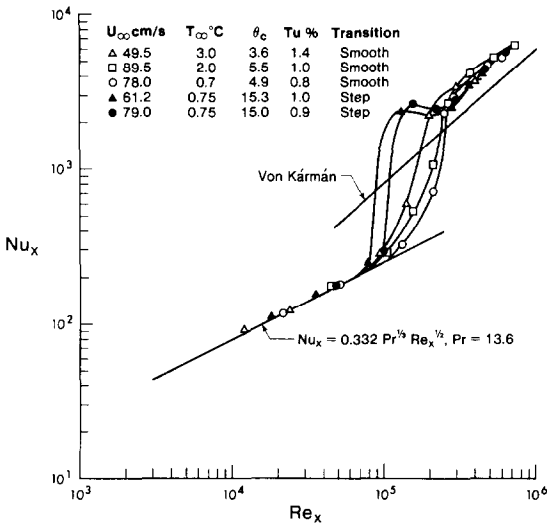


FIG. 9. Summary of the Nusselt number variation through the laminar, transition and turbulent regimes.

The transition from laminar to turbulent flow on an ice surface was found to be a most interesting and complex phenomenon. The factor that makes the flow transition on the ice more complicated than that on a flat “non-phase change” surface is one that often occurs in problems where both phase change and convection are involved. This is the mutual interaction of the shape of the ice surface, the fluid flow over the surface, and the heat transfer to the surface. In the case of the flow transition this mutual interaction results in two distinctly different transition modes occurring. Since the heat-transfer coefficient increases from laminar to turbulent values in the transition region the ice thickness on a constant temperature plate must decrease through the region. For the “smooth” transition mode the ice thickness decreases smoothly and the flow remains attached to the ice surface. It is speculated that turbulence generated in the first part of the transition is sufficient to maintain an attached flow through the region of unfavorable pressure gradient caused by the decrease in ice thickness.

In a second transition mode flow separation occurs and a “step” decrease in ice thickness results. Generally the “smooth” transition occurred on thin ice layers and the “step” transition occurred on thick ice layers. There was, however, a significant regime of flow conditions for which either mode could exist, depending on whether the ice thickness was being increased or decreased.

The onset Reynolds numbers (the Reynolds numbers at which turbulence produced a measurable increase in heat-transfer rates) on the ice surface were found to be lower than those for a flat plate. For a “smooth” transition the Reynolds number at which the heat transfer deviated significantly from the laminar theory was  $7 \times 10^4$  to  $1 \times 10^5$  as opposed to about  $3 \times 10^5$  for a flat plate. The difference was even greater for the “step” transition. Here the transition point was found to migrate upstream on the ice surface to a final steady state position which depended on an ice thickness parameter. Values of the Reynolds numbers at this “step” transition as low as  $3.9 \times 10^4$  were observed for thick ice layers.

The step transition mode has also been observed in ablation studies. For example in [1] which studied the penetration of a water jet into an ice block photographs of the cavity formed in the ice show a step change in the cavity diameter occurring at some distance from the jet impingement point. This abrupt change in diameter would now appear to be due to a step transition from laminar to turbulent flow. Since the ablation problems normally involve a thin film of liquid rather than a semi-infinite fluid flowing over a phase change surface a direct comparison of the ablation results with results of the present study can not be made. In fact, many of the phenomena observed in ablation studies have been attributed to the stability of and wave formation in this liquid film (3).

The shape of the ice surface in the transition regime had a large effect on the heat-transfer rates in the turbulent regime downstream of transition. Immediately downstream of transition, heat-transfer rates were 1.5 to 2.5 times greater than those for

turbulent flow on a flat plate. The effect of the step diminished further downstream; however, the heat-transfer coefficient remained significantly higher than that for a flat plate for Reynolds number at least up to  $7 \times 10^5$  into the turbulent regime.

From a review of the results presented in this paper it is apparent that the application to an ice surface of heat-transfer characteristics derived for a flat plate is a poor approximation. This applies to both the transition process and the heat-transfer calculation in the turbulent regime for some distance downstream of transition.

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#### PROFIL PERMANENT DE LA COUCHE DE GLACE SUR UNE PLAQUE A TEMPERATURE CONSTANTE, EN CONVECTION FORCEE: 2ème Partie—LES REGIMES DE TRANSITION ET TURBULENT

**Résumé**—Les mécanismes de transition entre le régime laminaire et le turbulent sur une surface de glace sont très différents de ceux sur une plaque plane. La forme de la couche de glace correspond au changement du coefficient de transfert thermique qui se produit dans le régime de transition. On observe deux modes de transition. Chaque mode est associé à une forme de profil caractéristique et à un nombre de Reynolds d'apparition du transfert thermique turbulent. Pour ces deux modes, ce nombre de Reynolds est nettement plus faible que pour une plaque plane. Pour quelques paramètres expérimentaux, on observe un phénomène d'hystérésis intéressant par lequel la couche permanente de glace peut prendre soit l'une, soit l'autre forme caractéristique, suivant la façon dont on approche l'équilibre. La décroissance de l'épaisseur de glace dans la région de transition a un effet important sur le transfert thermique dans le régime turbulent à quelque distance en aval de la transition.

#### DAS PROFIL DER STATIONÄREN EISSCHICHT AUF EINER PLATTE KONSTANTER TEMPERATUR BEI ERZWUNGENER STRÖMUNG—II. BEREICH DER ÜBERGANGS- UND TURBULENTEN STRÖMUNG

**Zusammenfassung**—Es wurde gefunden, daß der Übergang von laminarer zu turbulenter Strömung an einer Eisfläche sich im Vergleich zur ebenen Platte ganz wesentlich anders vollzieht. Der Einfluß der Eisoberfläche rührt daher, daß die Form der Eisschicht von Änderungen des Wärmeübergangskoeffizienten, wie sie im Übergangsgebiet auftreten, abhängt. Als Ergebnis dieser Wechselwirkung konnten zwei Arten des Übergangs beobachtet werden. Diese unterscheiden sich einerseits durch die Form des Eisprofils und andererseits in der Reynolds-Zahl, bei der turbulenter Wärmetransport einsetzt. In beiden Fällen ist diese Reynolds-Zahl wesentlich kleiner als für die ebene Platte. Für einige experimentelle Parameter ergab sich ein interessantes Hysterese-Phänomen, wobei die stationäre Eisschicht jeweils eine der beiden charakteristischen Formen annehmen konnte, je nachdem, von wo man sich dem Gleichgewicht näherte. Ebenfalls wurde beobachtet, daß die Abnahme der Eisschichtdicke, die im Übergangsgebiet auftritt, einen starken Einfluß auf den Wärmeübergang im turbulenten Gebiet in einer gewissen Entfernung stromabwärts vom Übergangsgebiet hat.



СТАЦИОНАРНЫЙ ПРОФИЛЬ СЛОЯ ЛЬДА НА ОБТЕКАЕМОЙ ПЛАСТИНЕ  
ПОСТОЯННОЙ ТЕМПЕРАТУРЫ. ЧАСТЬ II. ПЕРЕХОДНЫЙ И  
ТУРБУЛЕНТНЫЙ РЕЖИМЫ

**Аннотация** - Найдено, что переход от ламинарного режима течения к турбулентному на поверхности льда существенно отличается от процесса перехода на плоской пластине. Это различие вызвано тем, что изменяющийся в переходном режиме коэффициент теплообмена оказывает влияние на поверхность слоя льда, в результате чего наблюдаются два вида перехода, каждый из которых характеризуется особой конфигурацией профиля слоя льда и характерным переходным числом Рейнольдса. В обоих случаях значения переходного числа Рейнольдса были значительно меньшими, чем для плоской пластины. При некоторых рабочих параметрах наблюдалось интересное явление гистерезиса, при котором стационарный слой льда мог принимать любую из двух характерных конфигураций в зависимости от того, каким образом достигалось равновесие. Показано также, что уменьшение толщины слоя в области перехода оказывает большое влияние на интенсивность переноса тепла при турбулентном режиме на некотором расстоянии вниз по потоку от области перехода.